Mathematics teacher education

in Denmark and abroad

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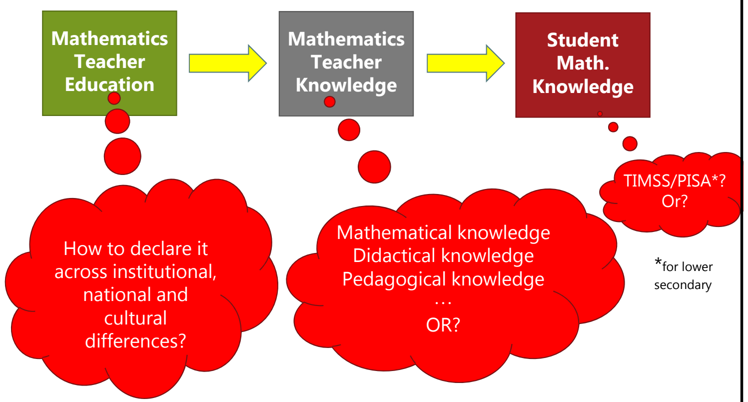
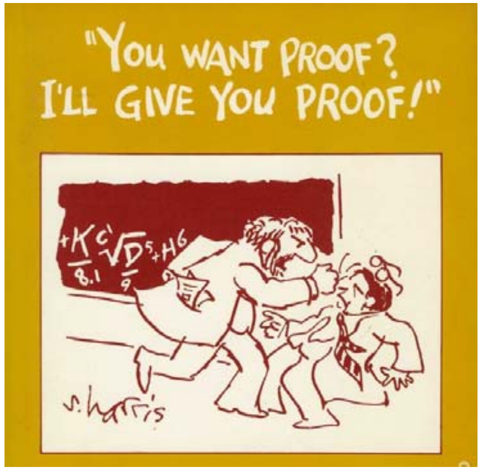
An impossible task?

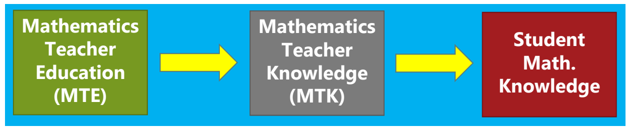
Demonstrate that/what differences in MTE matter, involves (at least naïvely)…



Alternative to knowledge…



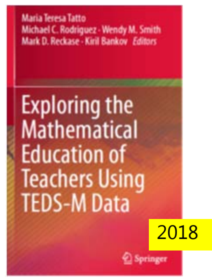
The main questions here



- What student achievements matter?
- What MTK matters for student achievement?
- How to model and investigate MTK to answer this? (at/near end of MTE)
- How to model and investigate MTE ?
- How to assess possible correlations between "measures" of the above?
- What can be inferred about MTE ?

Here: (some) main research findings and then focus on Denmark vs. other countries (even if DK has not participated in TEDS-M)

A main recent source:

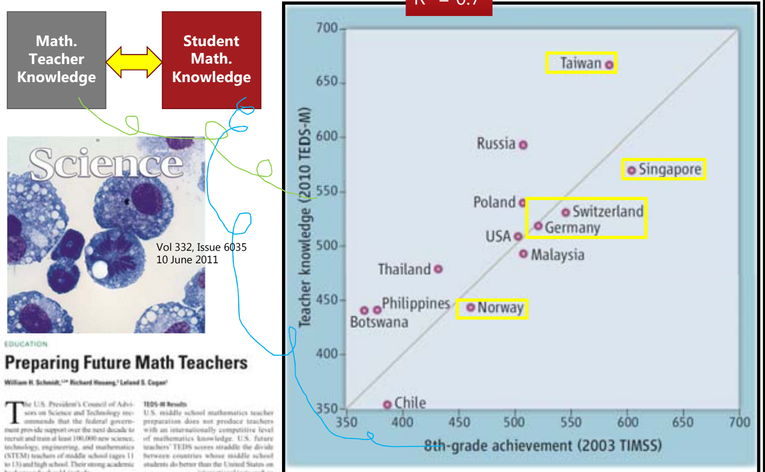


Some intriguing early findings…

- Teachers' having more (>6) academic math courses is not strongly correlated with higher achievement for their students (Begle, 1979; Monk, 1994; Ball et al., 2001)
- Mathematics majors do not automatically develop a deeper understanding of secondary level mathematics
  - *Completion of advanced college-level mathematics courses with passing grades does not imply mastery of the concepts of the K-12 curriculum* (Floden and Meniketti 2005)
- Quantitative studies of links between teachers' knowledge and students' learning show that *specific, practice-near teacher knowledge of mathematics* does matter (Thames 2009; Hill, Rowan and Ball, 2005)

Quandary: "research for policy" needs to analyse systemic effects of MTE, not individual ones based on one system…

$R^2 = 0.7$



Teacher preparation versus student scores in mathematics. TEDS scores for future middle school mathematics teachers versus TIMSS scores for eighth graders. See SOM.

How to classify and investigate MTK?

- Lehné (MSc-thesis, 2017): analysis of 3 major surveys
- Several theoretical frameworks, mainstream rooted in Shulman (1986) and distinguishing
  - Mathematical Content Knowledge (MCK)
  - Mathematical Ped. Content Knowledge (MPCK)
  - General Knowledge for Teaching (GKT)

  MKT (Ball et al) [MCK + MPCK]

  In TEDS-M, MCK and MPCK were measured for PS+LSS, while GKT was separately tested in 3 countries (D,US,TW).
- More fine-grained studies eg
  - Based on ATD/praxeologies: Durand-Guerrier, Yoshida, W (2009); Putra (2018), focusing on teachers *mathematical* and *didactical* praxeologies – notice praxis/logos distinction
  - Quantitative and qualitative studies of particular areas of MCK, MPCK or GKT (already many in 15th ICMI study, 2005)

TEDS-M design for measuring MCK and MPCK

MCK: based on TIMSS methodology and framework, ie adjusted to national curriculum, focusing on "content to be taught plus a bit more advanced". Categories : (1) number and operations, geometry and measurement, algebra and functions, data and chance, (2) knowing, applying, reasoning; (3) novice, intermediate, advanced.

MPCK framework:
- Content categories (essentially (1)-(3) above)
- MPCK-specific domain (cf next slide): (1) curricular knowledge, (2) planning for mathematics teaching and learning, (3) enacting mathematics for teaching and learning)

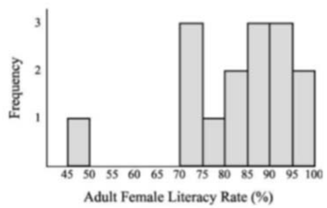
17 countries participated, including: Norway, Germany and the USA (public institutions). Research teams in each country. Elaborate design (see technical report, 2013). Finally, data from over 15,000 primary and over 9,000 lower-secondary future teachers and close to 5,000 teacher educators in 500 institutions of preservice teacher education.

MPCK-specific domain

Mathematical Curricular Knowledge[a]	Establishing appropriate learning goals Knowing different assessment formats Selecting possible pathways and seeing connections within the curriculum Identifying the key ideas in learning programs Knowledge of mathematics curriculum
Knowledge of Planning for Mathematics Teaching and Learning (pre-active)	Planning or selecting appropriate activities Choosing assessment formats Predicting[b] typical students' responses, including misconceptions Planning appropriate methods for representing mathematical ideas Linking the didactical methods and the instructional designs Identifying different approaches for solving mathematical problems Planning mathematical lessons
Enacting Mathematics for Teaching and Learning (interactive)	Analyzing or evaluating students' mathematical solutions or arguments Analyzing the content of students' questions Diagnosing typical students' responses, including misconceptions Explaining or representing mathematical concepts or procedures Generating fruitful questions Responding to unexpected mathematical issues Providing appropriate feedback

TEDS-M item for measuring MPCK - 1

The following graph gives information about the adult female literacy rates in Central and South American countries.



Suppose you ask your students to tell you how many countries are represented in the graph. One student says, "There are 7 countries represented."

Check one box.

	Right	Wrong
a) Is the student right or wrong?	☐1	☐2

b) In your opinion, what was the student thinking in order to arrive at that conclusion?

TEDS-M item for measuring MPCK - 2

A mathematics teacher wants to show some students how to prove the quadratic formula.

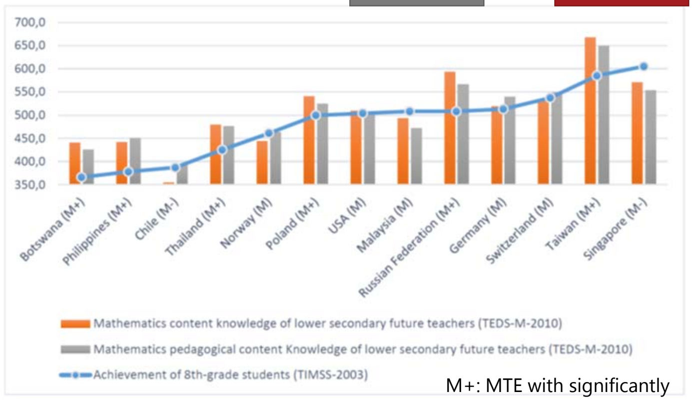
Determine whether each of the following types of knowledge is needed in order to understand a proof of this result.

Check one box in each row.

		Needed	Not needed
A.	How to solve linear equations.	☐1	☐2
B.	How to solve equations of the form $x^2 = k$, where $k > 0$.	☐1	☐2
C.	How to complete the square of a trinomial.	☐1	☐2
D.	How to add and subtract complex numbers.	☐1	☐2

Looking for patterns...

Mathematics Teacher Knowledge ⟷ Student Math. Knowledge



M+: MTE with significantly more focus om MCK

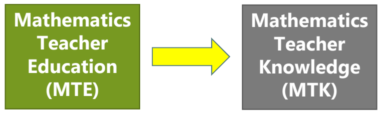
For instance…

- Very strong correlation MCK-MPCK (0,93); notice that MCK refers to mathematical knowledge *close to the target curriculum*…
- Overall strong correlation (about 0,70) between TIMSS and TEDS-M scores for MCK and MPCK. Causality – or third causes? Which influences which?
- In some countries the teachers are in some sense "better than the students". There is strong evidence that these *countries may recruit future teachers from high school graduates who were in the upper end of their country's eighth-grade TIMSS distribution*, and that *MTE in these countries is M+* (with less focus on GKT).
- Ratio of MCK/MPCK/GKT in TEDS-M top countries (Taiwan, Russia): 50/30/20; but Singapore tops TIMSS and is M-

How to classify and investigate MTE

Besides the measurement of MCK and MPCK, TEDS-M also included surveys on MTE (context, policy, institutions, curriculum). These may vary even within one country!

What are significant features for MTK or parts of it?



Let's consider… also for DK:

- Structure
- Components of MTE (in particular MCK/MPCK/GKT)
- Results from TEDS-M that may enlighten us … a bit

MTE in Germany and Denmark - structure

Country	Program-type	Consecutive/ Concurrent	Duration (years)	Grade span	Specialization
Germany	Teachers for Grades 1–4 with Mathematics as Teaching Subject (Type 1a)	Hybrid of the two	3.5 + 2.0	1–4	Generalist
	Teachers for Grades 1–4 without Mathematics as Teaching Subject (Type 1b)	Hybrid of the two	3.5 + 2.0	1–4	Generalist
	Teachers of Grades 1–9/10 with Mathematics as Teaching Subject (Type 2a)	Hybrid of the two	3.5 + 2.0	1–9/10	Specialist (in 2 subjects)
	Teachers for Grades 1–10 without Mathematics as Teaching Subject (Type 2b)	Hybrid of the two	3.5 + 2.0	1–4	Generalist
	Teachers for Grades 5/7–9/10 with Mathematics as Teaching Subject (Type 3)	Hybrid of the two	3.5 + 2.0	5/7–9/10	Specialist (in 2 subjects)
	Teachers for Grades 5/7–12/13 with Mathematics as a Teaching Subject (Type 4)	Hybrid of the two	4.5 + 2.0	5/7–12/13	Specialist (in 2 subjects)
Denmark	Prof. Bach. (B.Ed.)	Concurrent	4.0	1-6 4-9	Specialist (in ~3 subjects)
	M.Sc.	Consecutive	5.0+0.3	10-12	Specialist (in 2 subjects)

MTE in Norway and Denmark - structure

Country	Program-type	Consecutive/ Concurrent	Duration (years)	Grade span	Specialization
Norway	ALU with Mathematics Option	Concurrent	4	1–10	Generalist with extra math
	ALU without Mathematics Option	Concurrent	4	1–10	Generalist
	PPU	Consecutive	3 + 1 (or 5 + 1)	8–13	Specialist (in 2 subjects)
	Master's (MAS)	Concurrent	5	8–13	Specialist (in 2 subjects)
Denmark	Prof. Bach. (B.Ed.)	Concurrent	4.0	1-6 4-9	Specialist (in ~3 subjects)
	M.Sc.	Consecutive	5.0+0.3	10-12	Specialist (in 2 subjects)

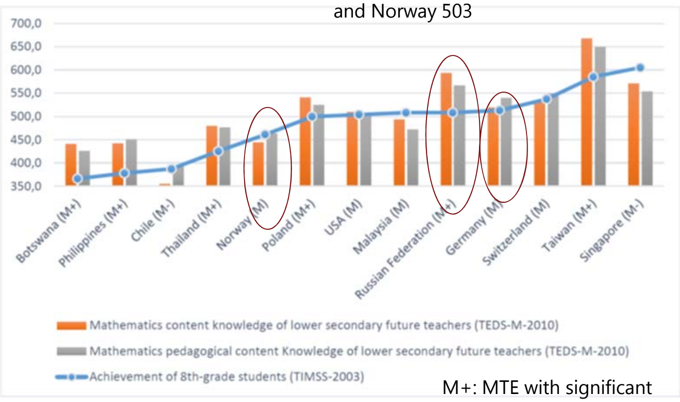
MTE in Denmark, Germany, Norway – contents in ECTS (legal minima, excluding practice)

	Denmark	Germany*	Norway
LS - MCK	40 (40%)	45 (38%)	60 (50%)
LS – MPCK		25 (21%)	
LS – GTK	60 (60%)	50 (42%)	60 (50%)
US – MCK	80 (74%)	80 (53%)	60 (57%)
US - MPCK	7.5 (7%)	25 (17%)	15 (14%)
US - GKT	20 (19%)	45 (30%)	30 (29%)

*Mean of minima across states (sd~10ects)

Analogues?

Denmark did not participate in TIMSS grade 8 2003, but in 1995 the Danish 8 graders scored 502, and Norway 503



M+: MTE with significant more "mathematics"

So… what matters?

Mathematics Teacher Education (MTE) → Mathematics Teacher Knowledge (MTK)

- TEDS-M took a closer look at the *top 10% LSS and PS programs* (in terms of graduates' MCK~MPCK) *from all 17 countries*. This identified programs from Poland (1), Russia (15), Taiwan (17) and the US (6).
- What contents do they meet?
- Typical distribution MCK/MPCK/GKT: **50/20/30** (similar to Germany US, but very different from other ones…)
- Benchmark "courses" (units identified by core subjects) identified as those taken by ≥80% of graduates in ≥ 90% of top programmes (next slides).
- Difficulty particularly with units on MPCK, given differing cultural and scholarly language (leaving some possibility that similarity of units might be overlooked)

Empirical requirements and electives identified from the international top performing lower secondary MTE programs…

Requirements	Electives
University Mathematics	**University Mathematics**
Beginning Calculus	Abstract Algebra
Calculus	Analytic Geometry
Multivariate Calculus	Axiomatic Geometry
Differential Equations	Number Theory
Linear Algebra	Set Theory
Probability	
Math Education	**Math Education**
Math Instruction	Math Standards
Observing/Analyzing Math Teaching	
School Mathematics	**School Mathematics**
Functions/Equations	Geometry
	Numbers
	Statistics

Note: Beginning Calculus, Calculus and Linear Algebra taken by 99% or more of the future teachers in these programs. Thus, *the most common courses in top programmes are absent in DK+NO LS MTE* (but present in the German ones).

Similarly, for primary school programmes…

Top 10% PS MTE programmes: Norway (1), Poland (17), Russia (11), Switzerland (1), Taiwan (10), Thailand (5), U.S. (4).

Empirical requirements and electives identified from the international top performing primary MTE programs

Requirements	Electives
University Mathematics	**University Mathematics**
Number Theory	Analytic Geometry
Probability	Axiomatic Geometry
Math Education	**Math Education**
Math Instruction	Math Standards
	Observing Math Teaching
School Mathematics	**School Mathematics**
Measurement	Functions
Numbers	Geometry

Discussion

- Top 10% programmes generally recruit top students. They may be “good” at the end no matter what programme they take (and perhaps better if it’s demanding).
- If you can’t recruit top students, different MTE contents might be better. But certainly not yield the same results (according to TEDS-M data).
- On the other hand, the end result *does matter* according to the graph we saw first… assuming we believe TEDS-M measures it rightly.
- At least, strong deviations from “benchmark” needs some justification as well…